FARADAY ROTATION EFFECT

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Abstract

Faraday Rotation is one of the historically most important experiments directly relating *Light and Magnetism*. Under this project we study the faraday rotation effect in laboratory as well as in astrophysical context. The measurement of faraday rotation angles was done with a new method, using which Verdet constants for different materials (SF-59, Distilled water and Benzene) at different wavelengths (650 nm and 543 nm) were also computed.

Acknowledgement

Contents

1   Introduction

In 1845, Michael Faraday *(Barr E.S., 1967)* discovered the first magneto-optical effect, called as *"Faraday Rotation Effect"*, which provided the first concrete evidence for a connection between magnetism and light, which is one of the historically important discoveries. It is observed that the plane of polarization of light rotates as a result of passage through a material in the direction parallel to an applied magnetic field. The rotation angle is called as *Faraday Rotation Angle.* This effect is different than the optical activity (Mason, 1968; Peterson 1975) shown by crystals. Faraday Rotation is an example of *Magnetic Birefringence.*

In this project we carry out investigation of Faraday Rotation Effect and study its application in Astrophysical context. It is observed that the Faraday Rotation angle (θ) is directly proportional to the magnetic field (B) and the length of the sample (L).

$$\theta = VBL$$

The proportionality constant (V) is called as Verdet constant.

We measure Verdet constants for Lead silicate glass, Distilled Water and Benzene at 650nm and 543nm wavelength Lasers. The detail classical theory explaining the Faraday Rotation Effect in transparent medium and in astrophysical plasmas, is also studied.

2   Theory

The Faraday effect has been discussed by many authers till date (Optik, Born; Polarized light, Collett; Fundamentals of Optics, Jenkins & white; Classical e-m Radiation, Marion and Heald). Although carefully studied from time to time but it was not modeled quantum mechanically untill 1960s. A simplified quantum mechanical tratment is also presented by David Van Baak (D. A. Van Baak, 1996).

The simplest classical treatment have been discussed very thoroughly by Edward Collett in his book 'Polarized Light'. Under this project we study only the classical theory for the Faraday Rotation Effect.

It is observed that when a Plane polarized light is incident on a optically thin material kept in a magnetic field parallel to the direction of propogation of light, the plane of polarization rotates. This is called as the *Faraday Rotation Effect.* The incident plane polarized light can be represented in the basis of two opposite handed polarized light namely right-circular polarized (RCP) and left cicular polarized (LCP) light (Optics, Hecht; Bickel and Bailey, 1985). When these two oppositely handed circularly polarized lights pass through the material kept in external magnetic field, they experience different refractive indices n_R and n_L. This causes a phase difference between the to rays, which results in rotation of the plane of polarization by an angle (θ), called as the 'Faraday Rotation Angle'. Hence it is directly proportional to the difference between two refractive indices and the distance traversed in the medium.

$$\theta = \frac{\pi d}{\lambda}(n_R - n_L)z \qquad (1)$$

where,

$$(n_R - n_L) \propto B$$

hence we can write,

$$\theta = VBz \tag{2}$$

where, V is a constant called as '*Verdet constant*'. The detailed analysis gives the expression for Verdet constant (V) as below,

$$V = \frac{4\pi Ne^3\omega^2}{m^2c^2(\omega^2 - \omega_o^2)} \tag{3}$$

where,

    V = Verdet constant

    n_R = Refractive index for RCP

    n_L = Refractive index for LCP

    ω_o = Resonance frequency

    N = Number density of electrons

    Hence θ is a function of wavelength (λ), magnetic field (B), the material used and the distance traversed (z). We measure the Verdet constants for different materials at different magnetic fields and wavelengths.

3 Experimental Setup

We use an apparatus designed by 'TeachSpin Inc.' (Jain and Tripathy, 1999; Loeffler, 1982). The apparatus consists of mainly, light source, solenoid (Magnetic field source), polarizer and analyzer polaroids and optical detector. The experimental arrangment is shown in Fig. 1 below.
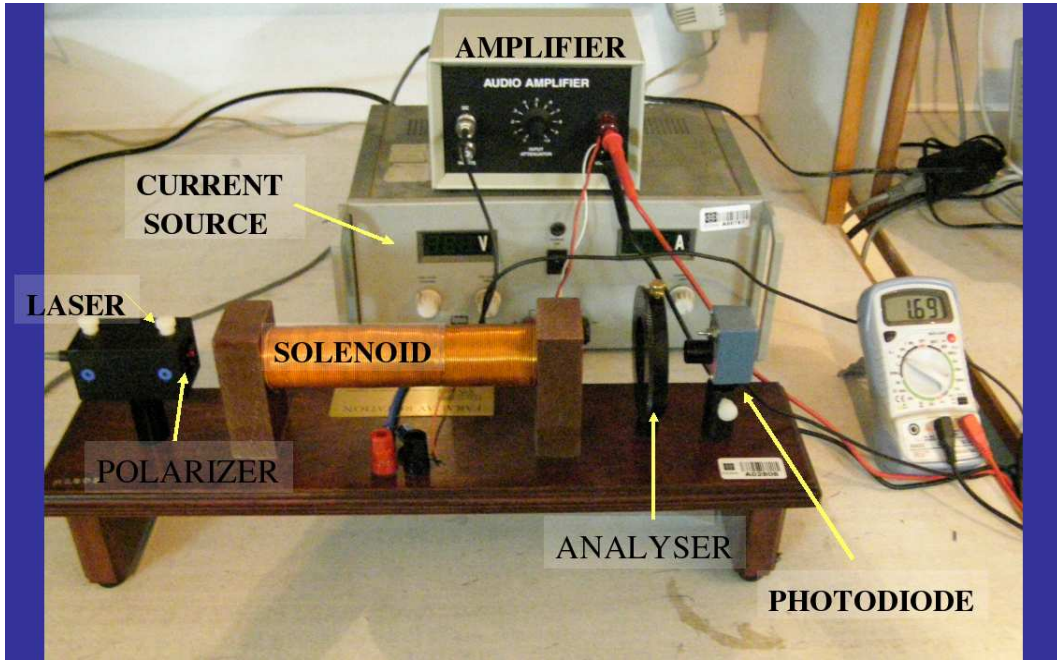

Figure 1: The experimental setup used for Faraday Rotation Angle measurement.

The laser light is passed through an polarizer, the output is ∼ 95% polarized and the power out put is about 3 mW. The polarized ligh is passed through the material (optically

thin) under study, which is kept inside the solenoid (magnetic field source), and then it is incident on the optical detector. The trasmitted intensity is measured by the optical detector.

3.1 Magnetic Field inside a finite solenoid

We must first measure the magnetic field inside the solenoid accurately. Because of the nonavailability of the Gaussmeter of required geometry, we could not measure the magnetic field experimentaly. Here we present the theoretical estimate of how the magnetic field would be inside a finite length and finite thickness solenoid as a function of location and applied current (I).

The solenoid used in the experiment has the following physical specifications.
Length = 15 cm
No. of turns(N) = 140 turns/layer
No. of layers = 10
Total no. of turns = 1400
Inner radius (a) = 0.88 cm
Outer radius (b) = 1.87 cm
Wire size = 18 double insulated
DC Resistance = 2.6 Ω
For all our further calculations we will be assuming these values.
Consider the magnetic field (B) due to a single loop,

$$B = \frac{\mu_o I r^2}{2(r^2 + z^2)^{\frac{3}{2}}} \tag{4}$$

Integrating it over a length L, the magnetic field due to a shell shape solenoid as a fuction of location (x_1, x_2) from the end points is

$$B(x) = \frac{\mu_o N I}{2L} \left[\frac{x_2}{\sqrt{r^2 + x^2}} - \frac{x_1}{\sqrt{r^2 + x_1}} \right] \tag{5}$$

And the direction is given by right hand thumb rule. Integrating this expression over a radial thickness from 'a' to 'b',

$$B(x) = \frac{\mu_o N' N I}{2L(b-a)} \left[x_2 ln \left[\frac{b + \sqrt{b^2 + x_2^2}}{a + \sqrt{a^2 + x_2^2}} \right] - x_1 ln \left[\frac{b + \sqrt{b^2 + x_1^2}}{a + \sqrt{a^2 + x_1^2}} \right] \right] \tag{6}$$

This equation gives the actual variation of magnetic field over the length of solenoid. We derive this equation as no standard text book contain such a complicated algebra. Although text books discuss the Magnetic field inside a soilenoid, which is obtained simply by appling Amphere's Law (Classical Eletrodynamics, Jackson),

$$\oint B.dl = \mu_o I_{enc}$$
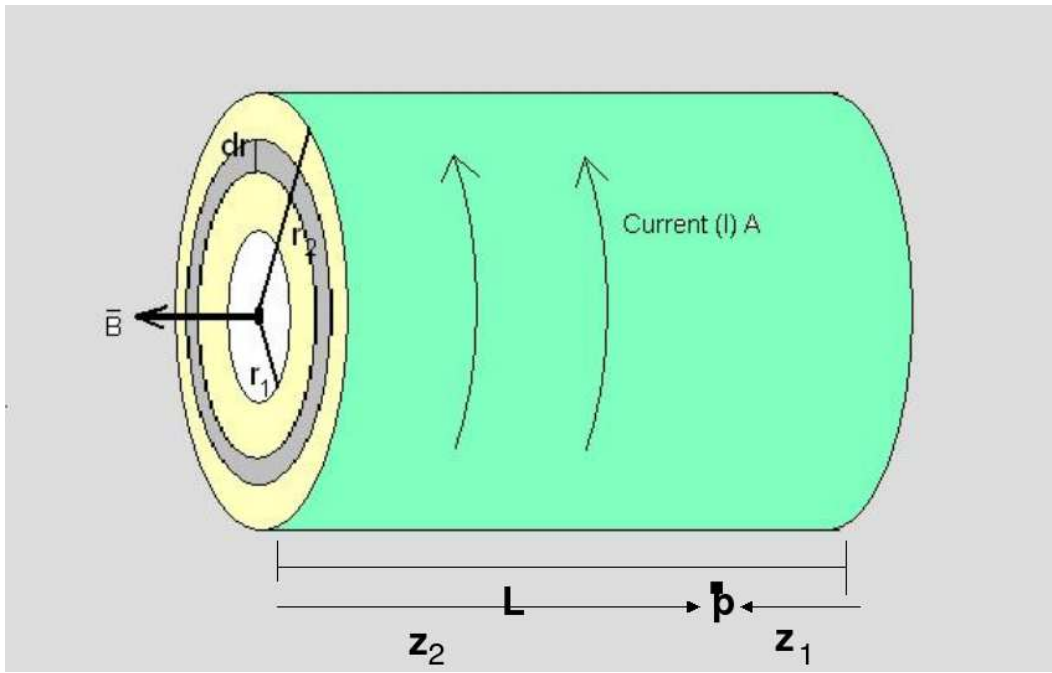
$$B = \frac{\mu_o N I}{L} \tag{7}$$

Figure 2: To obtain the exact magnetic field profile, integrate eq. 5 over small shells of thickness dr.

This simple equation tells us that the magnetic field inside a solenoid is constant but as we have just shown its not! (see our eq. (6)), this difference is basically due to the edge effect, where the magnetic field lines diverges. We wrote a computer program to see the magnetic field profiles given by above equations. The Fig. 3 shows the plot of eq. (5), eq. (6) and eq. (7) obtained.

We see that the plots of eq. (5) and eq. (6) shows only a slight variation, so we use the magnetic field profile given as eq. (6) and radius (r) as average radius (a+b)/2 for simplicity. The current source used is very stable (fluctuations less than 1%) and so is the magnetic field!

4   Verifying Malus's Law

When the polarizer axis is kept parallel with analyser axis, maximum light (I_o) passes through it. Since intensity is proportional to square of amplitude (Optics, Hecht), the transmitted intensity (I) when the relative angle between polarizer and analyser is 'ϕ' is,

$$I = I_o cos^2 \phi \tag{8}$$

Rotating the analyser through 360°, we measure the transmitted intensities, with and without magnetic field. And we observe the *cosine square* nature as expected. In the presence of magnetic field the incident plane of polarization of light undergoes faraday rotation (θ), so the whole Malus's curve is seen to be shifted by the same angle, see Fig. 4.
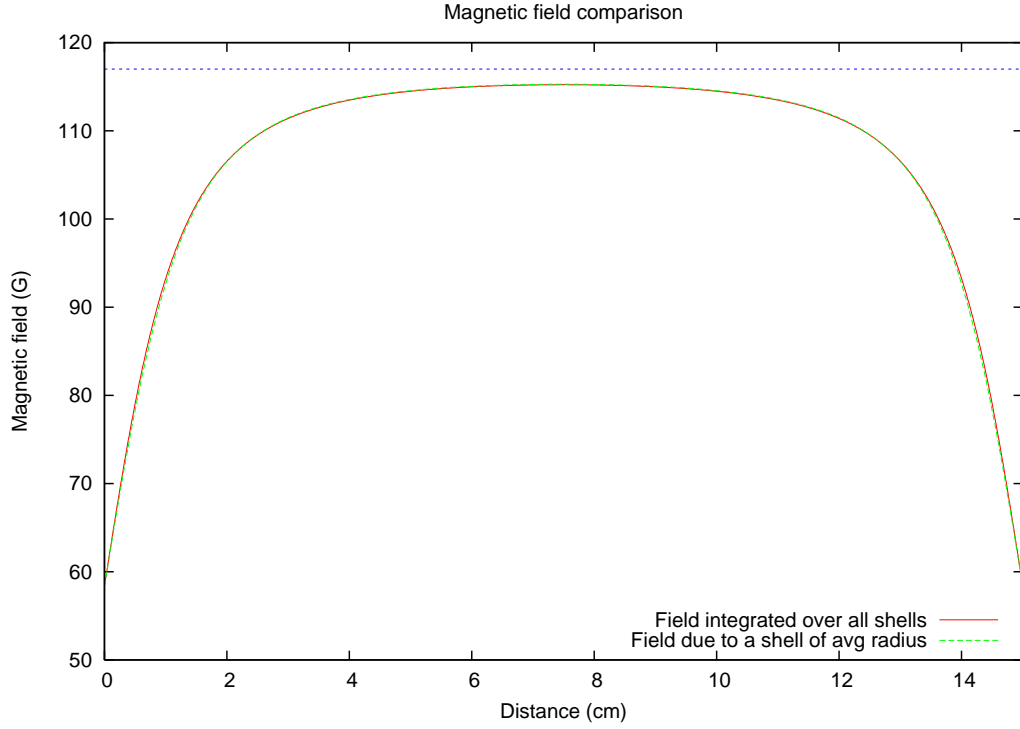
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Figure 3: Magnetic field profiles obtained from eq. (5) (Dotted green curve), eq. (6) (Solid red curve) and eq. (7) (Uppermost straight line), over the length of the solenoid.

5   Observing Method (using DC Magnetic Field)

The trasmitted intensity (I) at an relative angle ϕ of analyser and if the faraday rotation angle is θ, where θ is very small ($2^o - 3^o$), is given as,

$$I = I_o cos^2(\phi + \theta) \tag{9}$$

then,

$$\frac{\partial I}{\partial \phi} = I_o sin2(\phi + \theta) \tag{10}$$

Here we see the 'sensitivity' $(\partial I/\partial \phi)$ will be maximum when $\phi \sim 45^o$, i.e. the steeper region of the $cos^2\phi$ curve.

5.1   Existing Method

The well known method for measuring θ with DC magnetic fields is as follows. For maximum sensitivity the analyser is kept at 45° angle, and transmitted intensity is measured. This transmitted intensity changes when we turn ON the magnetic field. Now the polarizer is rotated such that the initial intensity is regained and the rotation angle is measured. Usually the rotation angle is around $2^o - 3^o$, but our analyser axis has least marks of only 5°, so we could not follow this method for accurate measurements of θ (Jain and Thripathy, 1999; loeffler, 1983). More accurate measurements can be done using AC magnetic fields (Wargreich and Christopher, 1997).
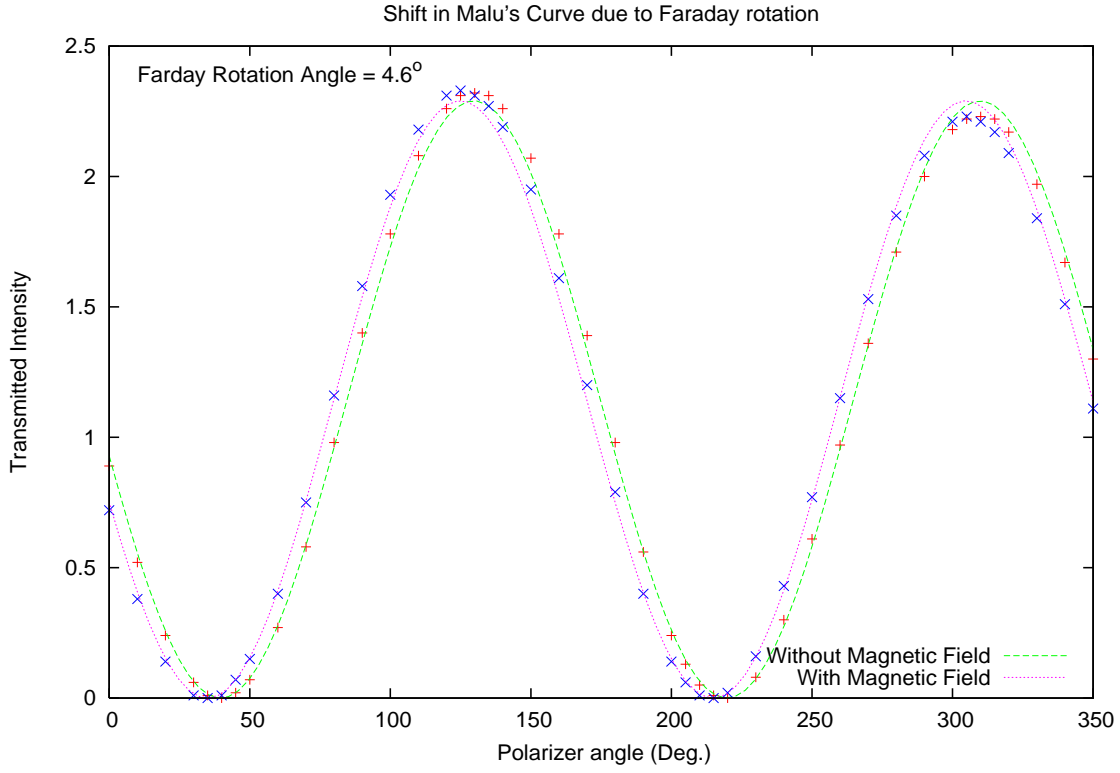
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Figure 4: Malus's curve with and without magnetic field.

5.2 New Method

We propose a new method for accurate measurement of θ. Initial (with magnetic field OFF) the analyser is kept at around 45° (α) to get maximum sensitivity. The measured intensity would be

$$I_x = I_o cos^2 \alpha$$

Now turn the axis exactly by 90° and measure the intensity again i.e

$$I_y = I_o sin^2 \alpha$$

We can then find out angle 'α' as,

$$\alpha = tan^{-1} \sqrt{\frac{I_y}{I_x}} \tag{11}$$

Now repeat the same procedure for the same angle with magnetic field ON. If the faraday roation is θ, the new relative angle would be $(\alpha - \theta)$, where the appropriate sign is taken by observing the sence of faraday rotation and the new intensity componets (see Fig. 5) would be

$$I_x^{'} = I_o cos^2(\alpha \pm \theta)$$
$$I_y^{'} = I_o sin^2(\alpha \pm \theta)$$

Hence we can calculate θ using eq. 11

$$\theta = \mp \left(\alpha - tan^{-1} \sqrt{\frac{I_y^{'}}{I_x^{'}}} \right) \tag{12}$$
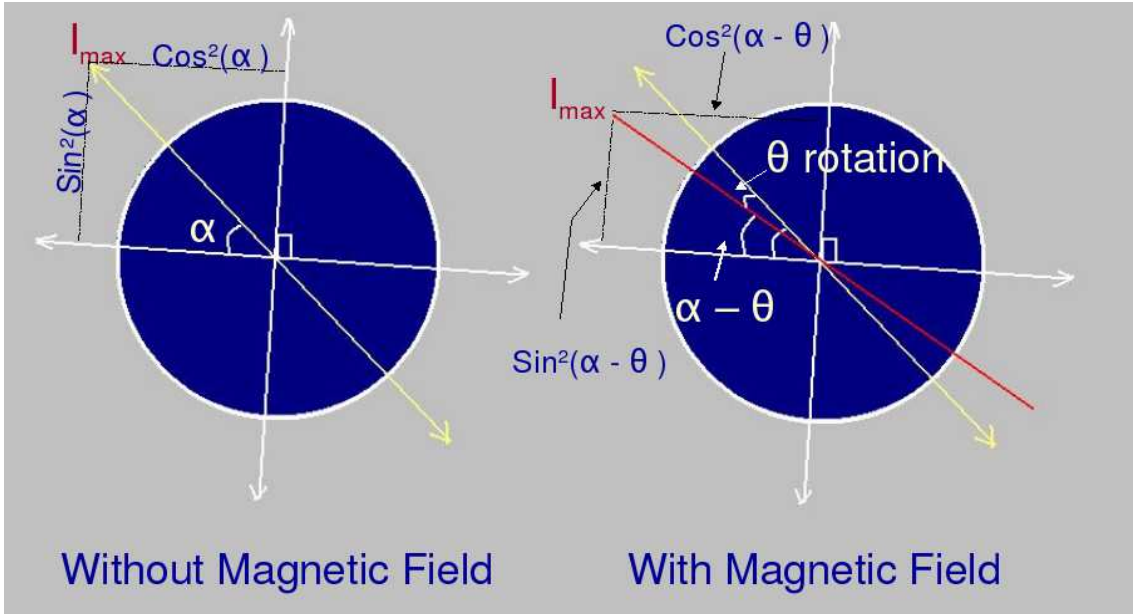
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Figure 5: The new method for measuring θ. We measure the intensity component wise with and without magnetic field. The two perpendicular axes shows two different positions of analyser for measuring the two components.

By this method the accuracy of measurement of θ is totally translated into how accurately we measure intensities, hence we can detect Faraday Rotation angles of the order of a degree.

This procedure is repeated for different analyzer angles ($\alpha \sim 30^o, 40^o$ and 50°) and for different magnetic fields. The average value puts a maximum error on the measurement of θ (the std. deviation is even less than the average error).

6   Observations

The new method of measuring θ, mentioned in the previous section was followed. Faraday Rotation angles for Distilled Water, SF-59 (Lead Silicate glass) and Benzene at different magnetic fields (few hundreds of Gauss) and wavelengths (650 nm and 543 nm) were measured successfully. Using the magnetic field profile given by eq. 5 we obtain the average magnetic fields inside the solenoid for different currents ranging between 1A to 5A. We plot this numerical estimate of magnetic field as a function of current (current fluctuations were less than 1%), see Fig. 6

As the magnetic field (B) inside the solenoid is not constant but is a function of position ($B(z)$), we need to modify eq. 2 as

$$\theta = V \int_L B(z).dz \tag{13}$$

where, V is Verdet constant.

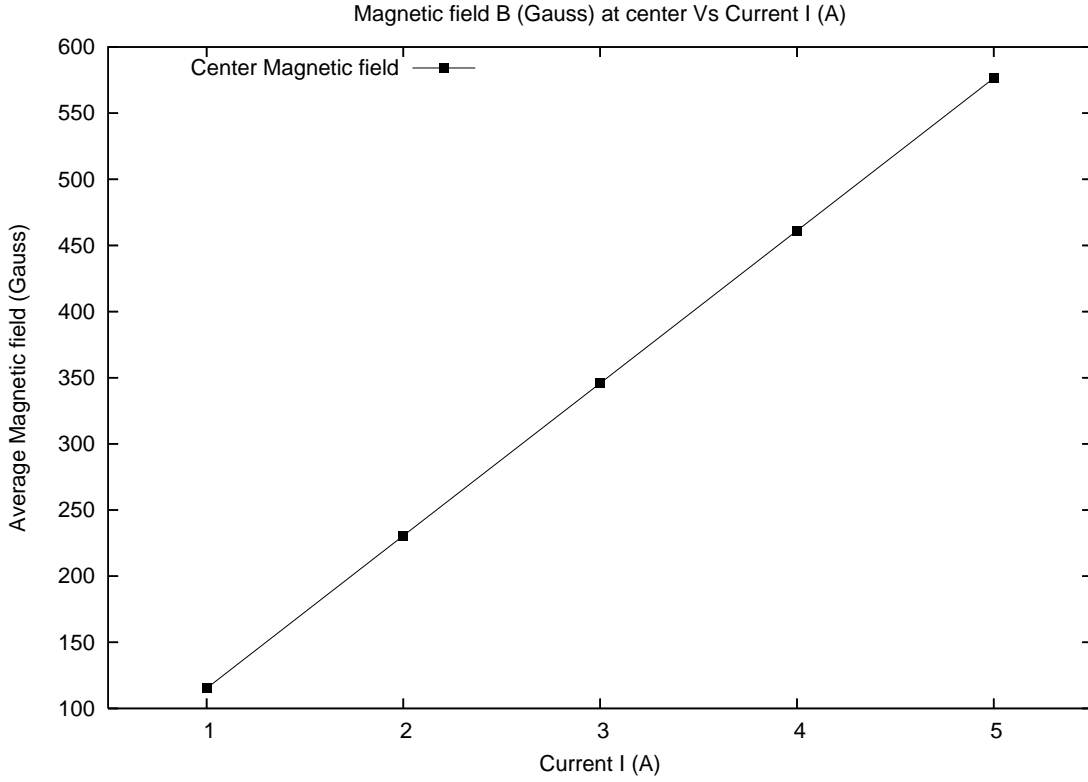It means the faraday rotation is a cumulative process over the length of sample. So we evelute the $\int_L B(z).dz$

Figure 6: Magentic fields for different currents in the solenoid.

6.1 Verdet constant of SF-59 (Lead Silicate Glass)

The SF-59 is specially developed Lead Silicate glass with very high refractive index (~ 2). The SF-59 sample extends over central 10 cm region inside the solenoid. We write a simple computer program to evaluate $\int_L B(z).dz$ for different currents (from 1A to 5A with steps of 1A).

We measure faraday rotation angle (θ) for the same currents and at two wavelengths 650 nm and 543 nm. The plot of θ Vs $\int_L B(z).dz$ is shown in Fig. 7. The least square fit to the observed data for 650 nm gives the Verdet constants (i.e. the slope) as V = 0.964 $\pm$ 0.045 $\times 10^{-3}$ $^o G^{-1} cm^{-1}$ and that for 543 nm gives V = 1.27 $\pm$ 0.12 $\times 10^{-3}$ $^o G^{-1} cm^{-1}$. The errors on measurement for green laser are quite larger than those for the red laser, its because of the fluctuations in the intensity of the green laser. But its clearly seen that the measured Verdet constant for SF-59 glass is greater at 543 nm than at 650 nm (within the error bars).

6.2 Verdet constant for Distilled Water

The water sample extends over 15 cm inside the solenoid. Similar to the above procedure, we numerically evaluate $\int_L B(z).dz$ for different currents (from 1A to 5A with steps of 1A). We measure faraday rotation angle (θ) for the same currents and at two wavelengths 650 nm and 543 nm. The plot of θ Vs $\int_L B(z).dz$ is shown in Fig. 8. The least square fit to the observed data at 650 nm gives the Verdet constants (V) as 0.184 $\pm$ 0.006 $\times 10^{-3}$ $^o G^{-1} cm^{-1}$.

The verdet constant mesurement at 543 nm could be done only for a single value
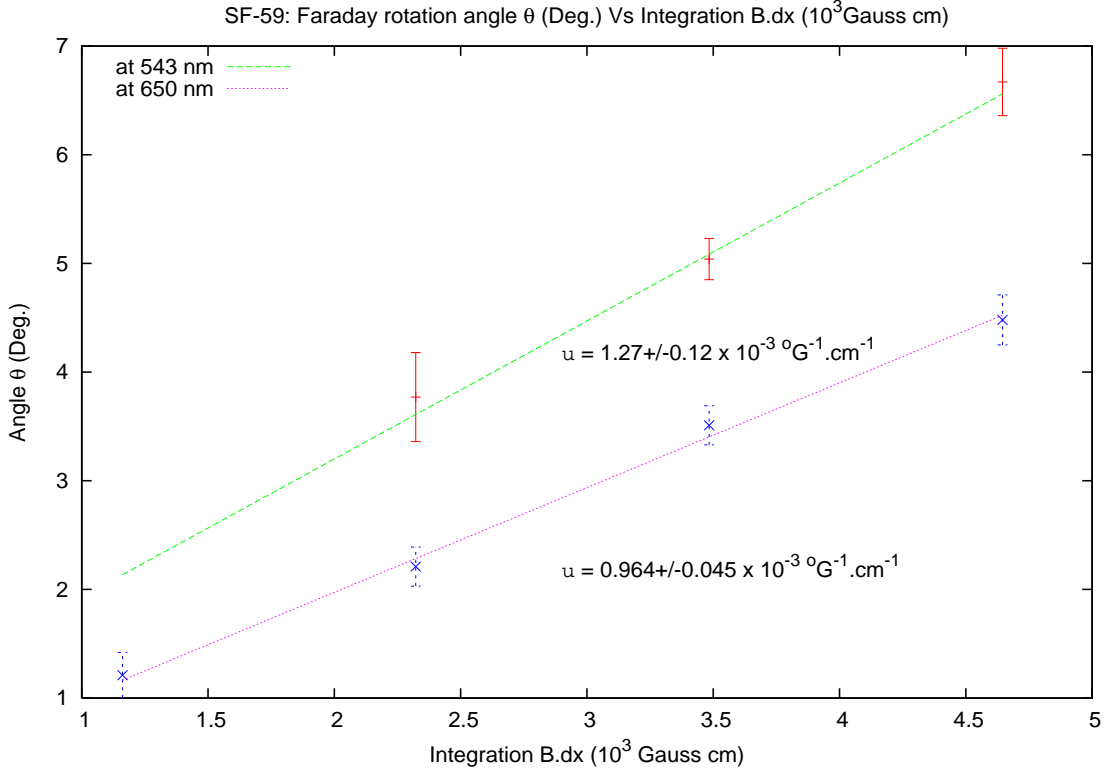
Figure 7: The slope of the two least square fit gives the Verdet constant. The lower line is at 650 nm and the upper one is at 543 nm wavelength.

of the magnetic field because of the fluctuations in the laser intensity and the verdet constant computed from it is $0.41 \times 10^{-3}~{}^oG^{-1}cm^{-1}$. The upper line (i.e. at 650 nm) is drawn considering the slope as the Verdet constant obtained with just a single set of measurements, so it has no errorbars. But it can be seen that the verdet constant at 543 nm is higher than that at 650 nm for water.

6.3   Verdet constant for Benzene

It was observed that at 650 nm the laser beam (of 3 mW power) was getting completely dispersed inside the Benzene column, due to this reason we could not measure the faraday rotation angles at 650 nm. At 543 nm laser beam of power 10 mW the Benzene column was observed to be expanding due to heating effect. It is also observed that benzene is very sensitive for change in temperature. Due to all these difficulties the faraday rotation measurement at 543 nm could be done for a short time and only for a single value of magnetic field. It is plotted over Verdet constants measured for other material, see Fig. 9. For comparison of measured Verdet constants for different materials and at different wavelengths, we combine all the plots together in Fig. 9.

Eq. 3 gives the wavelength dependance of the Verdet constant i.e.,

$$V = \frac{4\pi N e^3 \omega^2}{m^2 c^2 (\omega^2 - \omega_o^2)} \tag{14}$$

Here we plot the measured Verdet constants over two different wavelengths in Fig. 10. The line joining the points is *not* a fit, but its merely for the convenience of the reader
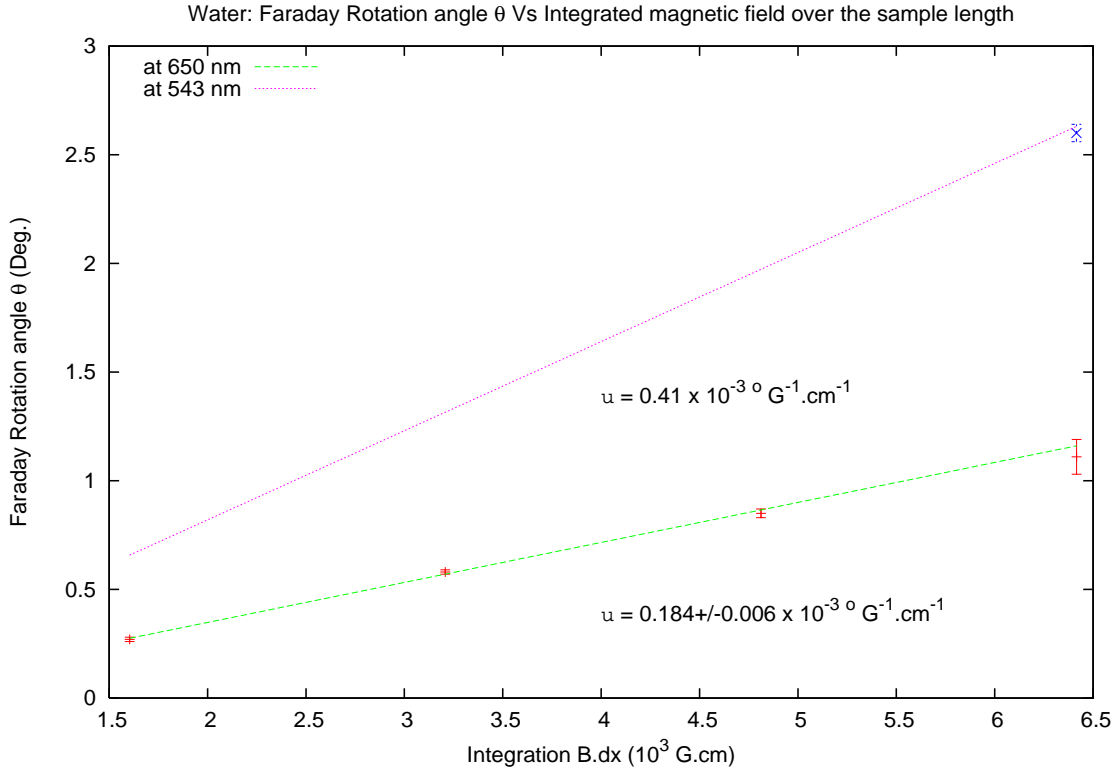
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Figure 8: The slope of the two least square fit gives the Verdet constant. The lower line is at 650 nm and the upper one is at 543 nm wavelength. The slope of the upper line is estimated only from a single magnetic field observations.

to distinguish between different samples. But it shows the nature of change in Verdet constant towards the lower wavelengths. It can be seen from eq. 14 that we can choose a frequency close to (but not very close) the resonance frequency to obtain very high Verdet constant.

7   Faraday Rotation in Astrophysical Laboratory

The Faraday Rotation Effect is also observed in astrophysical situations. In the very similar manner as in the laboratory when polarized light passes through a material kept in external magnetic field undergo faraday rotation, when the polarized light from a star with high magnetic field, passes through the interstellar medium (i.e. plasma) undergoes faraday rotation (Polarized light, Collett; Radiation Processes in Plasmas, Bekifi; Govoni and Feretti, 2006).

But for plasma the faraday rotation angle $\triangle\Psi$ is given by,

$$\triangle\Psi = \frac{e^3\lambda^2}{2\pi m_e^2 c^4} \int_0^L n_e(l)B_\parallel(l).dl \tag{15}$$

Hence it is proportional to λ^2 i.e. larger the wavelength of the light more will be the faraday rotation.

If the intrinsic angle of polarization of the light is Ψ_{int}, then the light we observe after
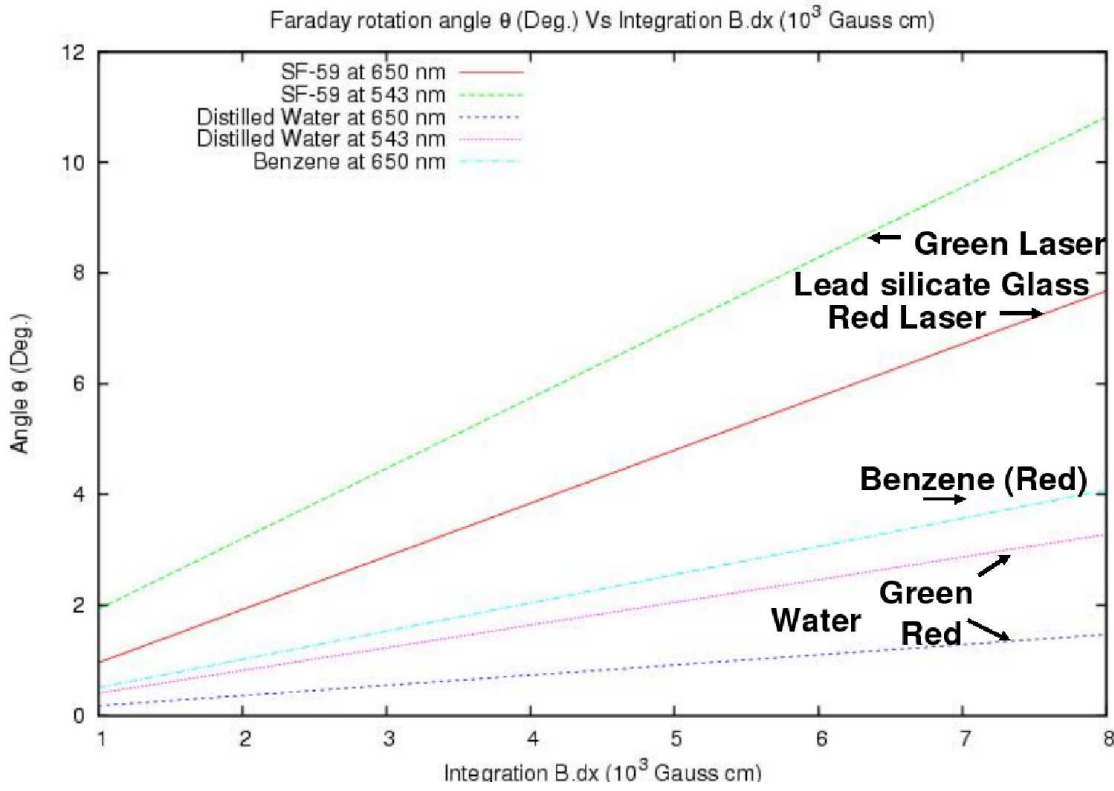
Figure 9: The slopes represents Verdet constants. It is seen that the verdet constant (the uppermost lines) for SF-59 is highest among Water and Benzene.

the cumulative faraday rotation from the source upto us is Ψ_{obs},

$$\Psi_{obs} = \Psi_{int} + \triangle\Psi$$

i.e.

$$\Psi_{obs} = \Psi_{int} + \frac{e^3\lambda^2}{2\pi m_e^2 c^4} \int_0^L n_e(l)B_\parallel(l).dl \tag{16}$$

where,
$n_e(l) = $ Plasma density as a fuction distance.
$B_\parallel(l) = $ Magnetic field (as a function of distance) component along the line of sight.
$\lambda = $ Observing wavelength
L = The distance between the source and observer
i.e.

$$\Psi_{obs} = \Psi_{int} + \lambda^2 RM \tag{17}$$

where,

$$RM = \frac{e^3}{2\pi m_e^2 c^4} \int_0^L n_e(l)B_\parallel(l).dl \qquad (rad/m^2) \tag{18}$$

i.e.

$$RM(rad/m^2) = 812 \int_0^L n_e(cm^{-3})B_\parallel(\mu G).dl(kpc) \qquad (rad/m^2) \tag{19}$$
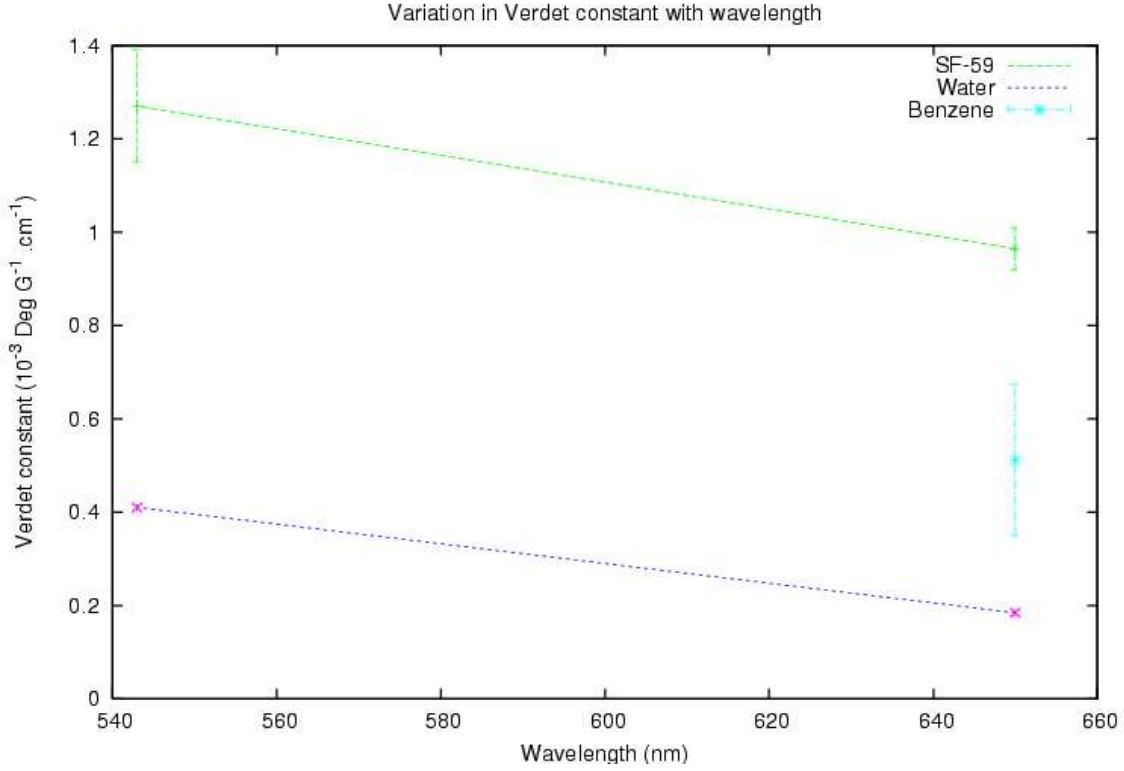
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Figure 10: Plot of Verdet constants measured Vs wavelength. The line joining the points is *not* a fit, it just connects the two observed points (of a single sample).

The quantity RM is called as '*Rotation Measure*'. In astrophysical situations it is more convenient to talk in terms of *Rotation Measure* instead of Magnetic field (B). It is in general very difficult to extract the magnetic field (B) (Clarke, 2004; Ruzmaikin and Sokoloff, 1979). But if the plasma density function $n_e(l)$ is known then it is possible to compute the value of source magnetic field.

The value of RM is easy to compute from eq. 17, from plot of observed Ψ_{obs} vs λ^2, the slope gives value of 'RM' and the Y-intercept gives the intrinsic polarization angle Ψ_{int}. The values of $RM's$ for different astronomical objects have been obtained by this method by many authors till date.

8   Conclusions

Under this project we study the Faraday Rotation Effect in detail. We also measure the faraday rotation angle with the new method and compute the Verdet constants for different material such as SF-59, Water and Benzene at 650 nm and 543 nm successfully. The faraday rotation effect in astrophysical context was also studied, but due to time contraint it was not possible to go into details.

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